

# THE PROBABILITY OF MAGNETIC OR ELECTROMAGNETIC DETECTION OF A 55-GALLON DRUM AS A FUNCTION OF LINE AND STATION SPACING

by

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## ABSTRACT

A simplified elliptical footprint of a 55-gallon drum for electromagnetic and magnetic detection as a function of depth is defined. The signal-to-noise ratio for field measurements is used to scale this ellipse. For a given ellipse, the probability of detection at various line and station spacings is calculated.

Representative numbers indicate that with a line spacing equal to the major axis of the ellipse and a station spacing equal to the minor axis, the probability of detection of a 1:2 (minor:major axis) ellipse by at least one station is 0.66, but only 0.12 for the recording of two anomalous values.

Compound probability enters the problem if more than one target is present. If location rather than detection is the goal, more than one anomalous station is required. If the number of stations is proportional to cost, location or characterization will certainly increase the costs. This paper provides information to disqualify specifications tendered by procurement departments who often request badly aliased surveys.

## INTRODUCTION

This paper offers one practical solution to the problem of survey design-i.e. the selection of line and station spacing given a hypothetical target. The near-infinite number of variants of the problem are indicated, but not solved. Some, but not all, of the philosophical questions are explored.

The first section indicates the solution to a simple problem which illustrates the probability technique. The mathematical problem for ellipse detection is then formulated the solution typically includes the numerical evaluation of an elliptic integral.

The practical aspects of application of this solution are then explored. Detection, location and characterization vs. signal-to-noise are the topics of discussion in this section. The Results section gives a few, practical numbers from the application of this methodology. A discussion section then discusses some of the numerous philosophical questions innate in this analysis.

The concluding section asks more questions than it answers, including those of cost-effectiveness, strategies, and areas of additional required work.

## PROBABILITY

The calculation of the probability of finding something can be a simple calculation if formulated in the following manner: Let an object be lost within an area A. Searches, which cover a portion of the area, S, will have a probability of success of:

$$P(\text{finding the object}) = S/A.$$

This analysis assumes that the probability of occurrence is uniform (your auto keys could be in the refrigerator) and that the definition of finding is unequivocal.

Let us define a more complicated problem based on Buffon's Needle Problem. (The original problem was not stated as a detection problem but is so recast here). What is the probability of detection (touching) of a needle of length  $a$  be dropped on a grid of parallel observation lines spaced a distance  $D$  apart, where  $D$  is greater than  $a$ . Variables are defined,  $X$  and  $TH$ , which are the distance of the center of the needle from the nearest line and the angle that the needle makes with a perpendicular to that line. The variable  $X$  is uniformly distributed in the region of  $0$  to  $D/2$ . The variable  $TH$  is uniformly distributed in the region  $0$  to  $2\pi$ . The needle is detected if

$$\left| \frac{a}{2} \cos TH \right| < X$$

Analogous to the simple object problem, the probability of detection for a given  $TH$  is

$$\frac{\frac{a}{2} \cos TH}{D/2} \text{ or } \frac{a \cos TH}{D}$$

Compound probability indicates that the probability of two events is the product of their separate probabilities. The probability that  $TH$  is in any given interval  $dTH$  is

$$\frac{dTH}{2\pi}$$

so the total probability of detection of the needle can be formulate as:

$$\int_0^{2\pi} \frac{a \cos TH}{D} \cdot \frac{dTH}{2\pi} = \frac{4a}{2\pi D} \int_0^{\pi/2} \cos TH \, dTH = \frac{2a}{(\pi D)}$$

A similar problem for an ellipse can be formulated with a step increase in the amount of algebra and a numerical integration of the resulting elliptic integral.

Another complicating issue is a consideration of the problem of a finite station spacing along the lines. The above analysis assumes continuous observation along the lines. By formulating the equations for the length of line that lies within the ellipse and by dividing this length by the station spacing, the probability of detection by  $N+1$  stations can be derived based on the probability of detection by  $N$  stations. This process can be continued down to the detection by only one station.

### DETECTION, LOCATION AND CHARACTERIZATION

For purposes of this paper, detection is defined as the location of one anomalous point on the buried object. The detection of an anomaly based on a single point puts pressure on the signal to noise ratio. An unequivocally anomalous point is required.

Location is also a nebulous concept. For purposes of this paper, location is defined as 4 anomalous points on the body. This definition is strictly a judgement call and assumes that some information about the shape ;and size of the anomaly is known.

Characterization is defined as a geophysical interpretation. That is, sufficient data are available that the size, depth, orientation, and physical contrast (density difference or magnetization) can be determined. A good rule of thumb for geophysical inversion is that two or more observations are required for each parameter determined. Thus 6-20 anomalous points may be required for characterization. Recent advances in UXO technology, generally call knowledge-based- systems, are an attempt to reduce the number of observations by use of a priori knowledge about the type of munitions most likely present.

## ANOMALIES OF SPECIFIC BODIES AND SIGNAL-TO-NOISE RATIOS

The ubiquitous 55-gallon drum is a well studied target. Depending on its orientation, it will have a magnetic half-power width (in profile) equal to its depth. The ellipticity of the anomaly is not extreme, generally about 1:1.5 (minor:major).

Even the vertical drum exhibits some ellipticity in its anomaly pattern due to the inclination at U.S. latitudes. As the amount of steel and its susceptibility is variable, the peak value of the anomaly can vary from 500 to 1500 nanoTeslas (nT).

In an urban environment, the noise of a conventional magnetics survey may vary from 20 to 500 nT. If a pessimistic approach is taken with respect to signal-to-noise ratios, a drum may be only detectable within an ellipse whose long axis is equal to its depth. This analysis is simplified and is presented here only for purposes of calculation of the probabilities.

Electromagnetically, the problem is equally complicated. If the EM-31 is used as a tool, an anomaly of 2 ppt for in-phase values may be required to be clearly anomalous in an urban environment. This value is reached some distance from the drum depending on the orientation of the drum and the instrument. This distance can also be simplified as being equal to an ellipse equal to the depth of burial of the drum though, for deeper drums, the anomalous values may decrease below 2 ppt. Depending on the orientation of the drum, this anomalous footprint may be elliptical in shape with an ellipticity ratio of about 1:2.

### RESULTS

Once the ability to calculate the probability of detection or location by a given observation scheme is in place, it is interesting to calculate a wide variety of probabilities for various bodies. In order not to confuse the reader, only two simple cases will be treated here.

In both cases, the distance between observation lines will be set equal to the major axis of the ellipse sought. Two cases will be considered, ellipses with ratios of 1:2 and 0.7:1. The probabilities of detection (one or more anomalous points) and location (4 or more anomalous points) will be considered. The following table gives the results:

TABLE 1.  
DETECTION AND LOCATION AS A FUNCTION OF STATION SPACING

ELLIPTICITY RATIO	STATION SPACING AS PORTION OF LINE SPACING	PROBABILITY OF DETECTION	PROBABILITY OF LOCATION
1:2	0.0	77%	
	0.05		75%
	0.1		50%
	0.2		12%
	0.5	67%	0%
	1.0	40%	0%
0.7:1	0.0	85%	
	0.05		84%
	0.1		77%
	0.2		37%
	0.5	80%	0%
	1.0	55%	0%

Note that a 0.0 station spacing implies continuous observations.

## DISCUSSION

From the few cases presented here, a couple of generalizations can be made. If one wants to locate bodies (numerous anomalous points) one must have a lot of stations along the lines. Newer instruments, specifically some of the magnetometers, have the memory and short cycle times to approach continuous observations. Secondly, it is clearly beneficial to have a station spacing smaller than the line spacing as the additional stations greatly increase the probability of detection. Often, the cost of additional stations along the survey lines is not high.

As the probabilities of location are not particularly high for these cases, a strategy of detection and the location by a grid of greatly reduced station and line spacings is indicated.

Philosophically, one must come to grips with the concept of probability. Dr. Phil Romig in several papers has indicated that the usual environmental tolerance is near zero for nondetection. The values used above for threshold values are pessimistic and certainly low-noise surveys will produce a larger footprint for a drum. The treatment of depth is not satisfactory as shallow bodies will often have a small footprint which gets larger for deeper depth of burial. At some point, this footprint begins to shrink as the signal from the deeper body decreases. The problem of characterization is not treated here

## CONCLUSIONS

This discussion was instigated by the receipt of an RFB (Request for Bid) for a hazardous waste site which specified magnetic and EM-31 observations with spacings of 50 feet. The specified targets were drums, possible as few as one in any particular group. Protests to the procurement department issuing the RFB indicated that they were bound by "contractual agreements" to perform the survey to these specifications. To my knowledge, the survey was performed, but not by my employer.

## REFERENCES

- Butler, David, 1967, The Geometric Probability of Ellipse Detection, paper presented at the 37th Annual International Society of Exploration Geophysicists Meeting, referenced in Geophysics, Vol. 32, No. 6.
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