15. MATHEMATICAL ALGORITHMS

15.1   STANDARD ERROR OF THE MEAN (SEM)

The standard error of the mean or SEM is the standard deviation divided by the square root of the number of cycles or bursts used to obtain the data:

$$SEM = \frac{\sqrt{\dfrac{\sum\limits_{j=1}^{N} x_j^2}{N} - \left(\dfrac{\sum\limits_{j=1}^{N} x_j}{N}\right)^2}}{\sqrt{N}}$$

$where: x_j = \arctan\left[\dfrac{imaginary_j}{real_j}\right]$ for frequency domain programs

$x_j = \dfrac{Window\_Integral_j}{V_{pj}}$ for time domain programs

$x_j = \dfrac{Window\_Voltage_j}{Current}$ for TEM programs

and: $N =$ the number of cycles acquired

	Units
SEM (TDIP) $= $ SEM x 1000 x $\dfrac{0.116875}{frequency}$	Milliseconds
(RPIP) $=$	
SEM (CSAMT) $=$ SEM x 1000	Milliradians
(CR) $=$	
SEM (TEM) $=$ SEM	Scaled units in V/A

15.2 3-POINT DECOUPLING

QUADRATIC DECOUPLING EQUATIONS

Harmonic Frequencies

1. First, third, fifth harmonics   (0.1,0.3,0.5 Hz; 0.125,0.375,0.625 Hz; etc)

$$\phi_c = (15 \phi_1 - 10 \phi_3 + 3 \phi_5) / 8$$

2. Third, fifth, seventh harmonics   (0.3,0.5,0.7 Hz; 0.375,0.625,0.875 Hz; etc)

$$\phi_c = (35 \phi_3 - 42 \phi_5 + 15 \phi_7) / 8$$

3. 0.1, 0.3, 1.0 Hz   (0.1, 0.3, 1.0 Hz)

$$\phi_c = (35 \phi_1 - 15 \phi_3 + \phi_{10}) / 21$$

4. First, third, fifth, seventh harmonics   (.1,.3,.5,.7 Hz; .125,.375,.625,.875 Hz; etc)

$$\phi_c = (35 \phi_1 - 35 \phi_3 + 21 \phi_5 - 5 \phi_7) / 16$$

Binary Frequencies

5. Three sequential binary frequencies   (0.125,0.250,0.5 Hz; 8,16,32 Hz; etc)

$$\phi_c = (8 \phi_1 - 6 \phi_2 + \phi_3) / 3$$

6. First, second, and fourth sequential binary frequencies   (0.125,0.250,1.0 Hz)

$$\phi_c = (48 \phi_1 - 28 \phi_2 + \phi_4) / 21$$

7. Four sequential binary frequencies   (0.125,0.250,0.5,1.0 Hz; 4,8,16,32 Hz; etc)

$$\phi_c = (8 \phi_1 + 2 \phi_2 - 5 \phi_3 + \phi_4) / 6$$

When operating in low resistivity environments with large dipole spacings, as is common with dipole-dipole or pole-dipole surveys, electromagnetic coupling is often an overwhelming factor in IP measurements. The Complex Resistivity (CR) program provides for automatic removal of electromagnetic effects by using a 3-point decoupling algorithm, which assumes that the IP effect at very low frequencies is relatively independent of frequency. This routine works well in moderately coupled environments. For extreme coupling environments (e.g., 10 ohm-meter ground and using 300 m dipoles), we recommend that full frequency CR be used to permit more precise coupling removal.

The formula used for 3-point decoupling in the GDP-32^{II} is as follows:

$$\phi_{3pt} = 1.875 \phi_1 - 1.25 \phi_3 + 0.375 \phi_5$$

ϕ_1 = phase at the fundamental frequency

ϕ_3 = phase at the third harmonic

ϕ_5 = phase at the fifth harmonic

15.3 A NOTE ON RESISTIVITY CALCULATIONS

Since received voltage magnitudes for RPIP and CR are calculated using Fourier transforms, the square-wave current that is used to calculate apparent resistivities must be multiplied by $4/\pi$ to get its equivalent Fourier amplitude. Therefore, the values of I (Tx Curr) used in Section 15.4 must be multiplied by $4/\pi$ for RPIP and CR to get the correct apparent resistivities.

Resistivity calculations for TDIP use the square-wave value of the received voltage during the on-time (V_p), so the square-wave current amplitude can be used directly.

15.4 RESISTIVITY CALCULATIONS

Five types of arrays are defined in the GDP-32^{II} programs for which resistivities are to be calculated. These are Dipole-Dipole, Pole-Dipole, Schlumberger, and Gradient.

The parameters that are manually entered by the operator, and the values that are measured by the GDP-32^{II} are bracketed [] in the description of parameters.

DIPOLE - DIPOLE
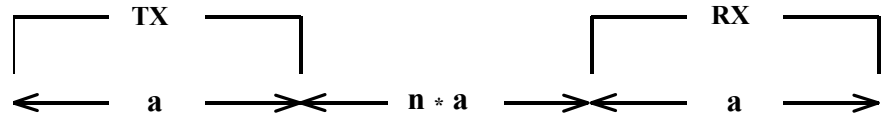
RHO = (π * V / I) * (a * n * (n + 1) * (n + 2))
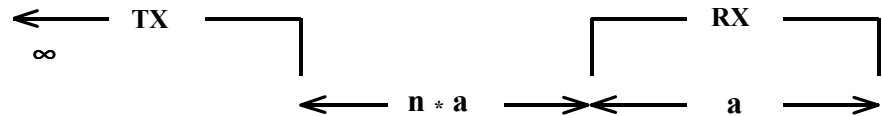Where: a = [A-SP]   = A-spacing in meters
         n = [N]   = N-spacing
         V = Received voltage in volts [Measured]
         I = 4/π * [Tx Curr] = Transmitter current in amperes * 4/π

POLE - DIPOLE

RHO = (π * V / I) * (2 * a * n * (n + 1))
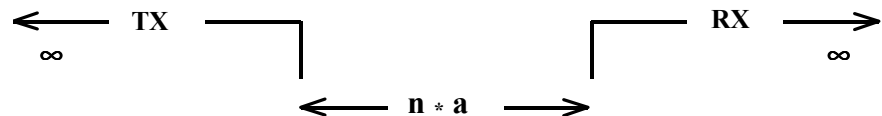Where: a = [A-SP]   = A-spacing in meters
         n = [N]   = N-spacing
         V = Received voltage in volts [Measured]
         I = 4/π * [Tx Curr] = Transmitter current in amperes * 4/π

POLE - POLE

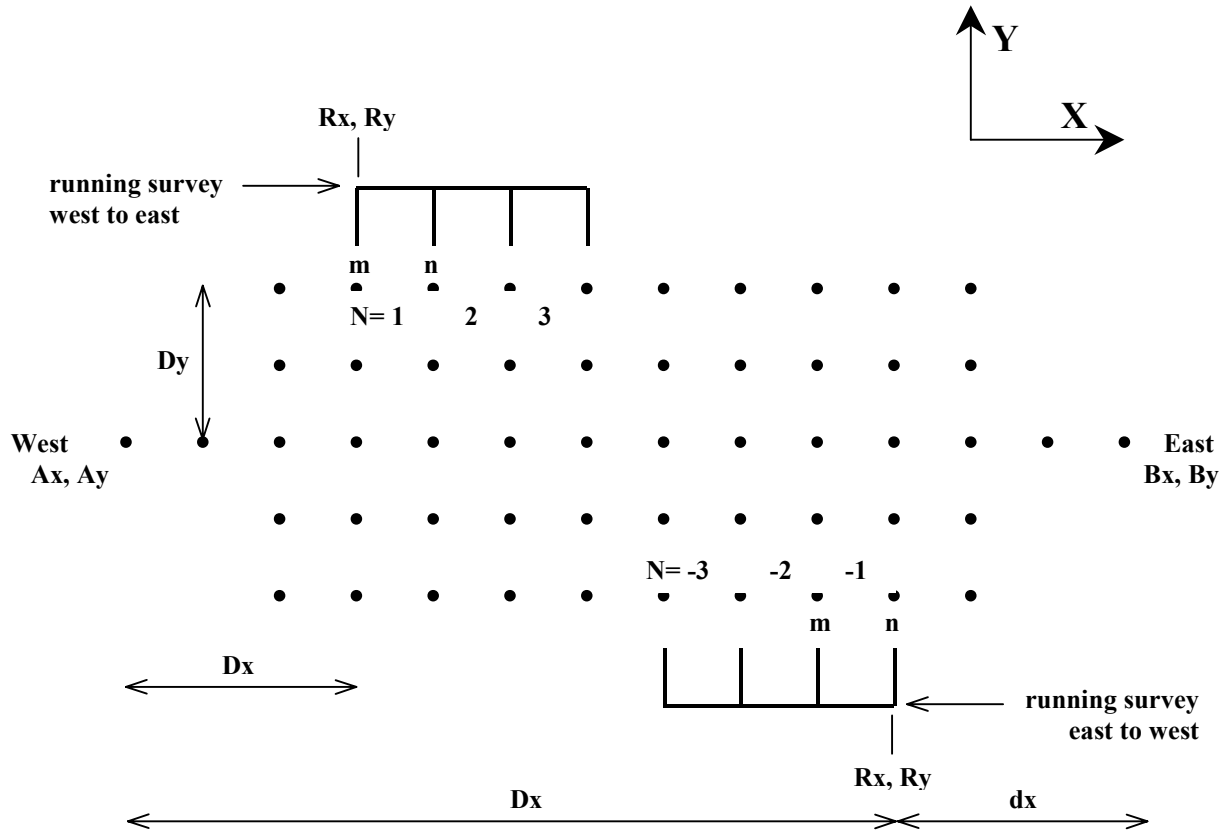**RHO = (π * V / I) * a * n * 2**
Where: a = [A-SP]   = A-spacing in meters (basic separation increment)
         n = [N]   = Number of A-spacings between electrodes
         V = Received voltage in volts [Measured]
         I = 4/π * [Tx Curr] = Transmitter current in amperes * 4/π

GRADIENT

where:

$$RHO = 2\pi * \frac{V}{I} * \cfrac{1}{\cfrac{1}{mA} - \cfrac{1}{mB} - \cfrac{1}{nA} + \cfrac{1}{nB}}$$

$m_1A \qquad = \surd(Dy^2 + Dx^2)$

$n_1A \quad = m_2A = \surd(Dy^2 + (Dx+N_{1*}a)^2)$

$n_2A \quad = m_3A = \surd(Dy^2 + (Dx+N_{2*}a)^2)$

$n_3A \qquad = \surd(Dy^2 + (Dx+N_{3*}a)^2)$

$Dx \quad = |Ax - Rx|$

$Dy \quad = |Ay - Ry|$

$dx \quad = |Ax - Bx| - Dx$

$m_1B \qquad = \surd(Dy^2 + dx^2)$

$n_1B \quad = m_2B = \surd(Dy^2 + (dx-N_{1*}a)^2)$

$n_2B \quad = m_3B = \surd(Dy^2 + (dx-N_{2*}a)^2)$

$n_3B \qquad = \surd(Dy^2 + (dx-N_{3*}a)^2)$

For these calculations, we are assuming numbers (coordinates) increase in value to the right and up, (to the east and north, for example).

GRADIENT ARRAY NOTES

The objective of this resistivity calculation scheme is to make it easy to input grid coordinates for gradient surveys.

CAUTION: The LINE number in Menu 2 has been changed to Ay in this program as the Y-coordinate of the AB transmitter dipole. Carefully read the following definitions:

a = [**A-SP**] = a-spacing (size of receiver dipole) in meters

V = Received voltage in volts [Measured].

I = 4/π * [**Tx Curr**] = 4/π * Transmitter current in amperes.

> The 4/π factor is included in the software.

Ax, Bx = Transmitter current electrode locations, A and B. Ax and Bx are input in Menu 1, in meters, and are coordinate locations.

Ay = By = Y-coordinate of the transmitter dipole in meters, which is input in Menu 2. Set to 0 if Tx dipole runs along the baseline.

AB = |**Ax - Bx**| = Transmitter dipole length in meters.

Rx and Ry are the coordinates of the first electrode in a receiver setup.

Rx = [**Rx**] = Coordinate parallel to the transmitter dipole of the first potential electrode. It will be the western-most electrode if running west-to-east; eastern-most if running east to west. If Ax is set to zero, Rx will be equal to Dx and will be the distance from Ax.

Ry = [**Ry**] = Coordinate perpendicular to the transmitter dipole of the first potential electrode. If LINE is set to zero, Ry will be the distance from the AB dipole and will be equal to Dy.

N = Electrode location (in multiples of a) with respect to Rx. This value is positive (+) if running in the direction of increasing coordinates, negative (-) if running in direction of decreasing coordinates. (This value is used as N-spacing in other programs.)

Summary of input parameters for resistivity calculations:

Ax, Bx : Menu 1, in meters. "X" Coordinates of the transmitter dipole.

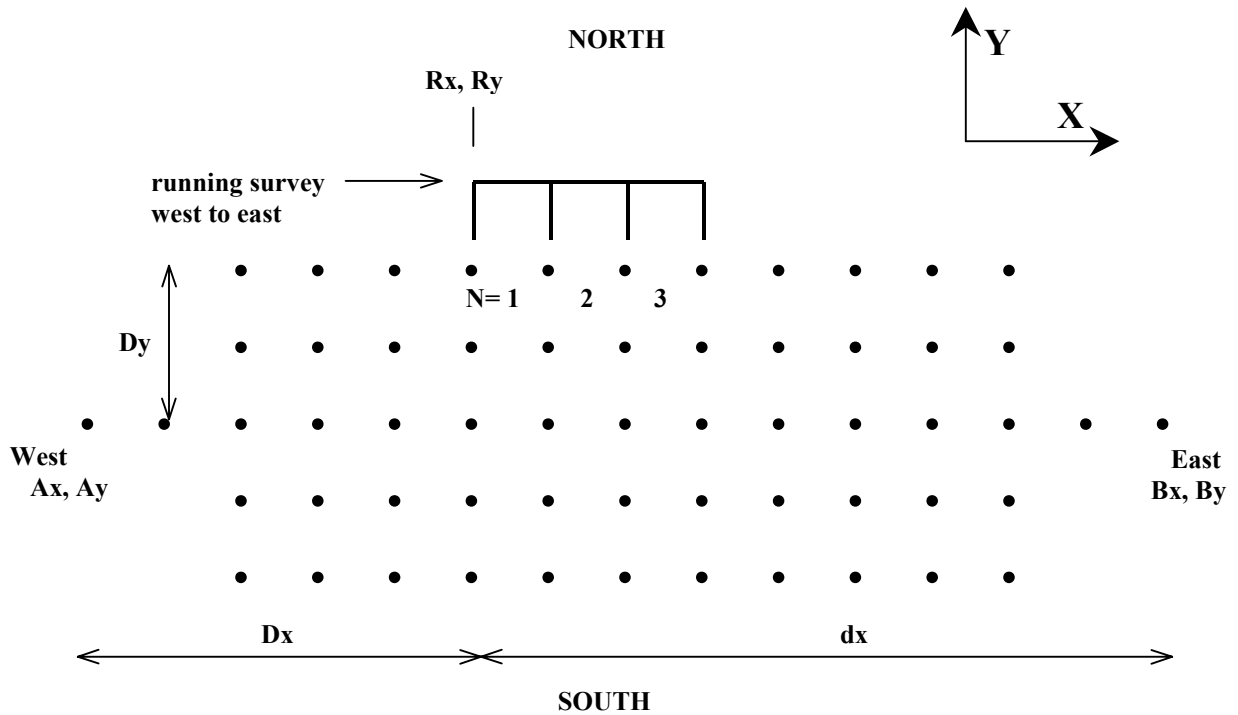Ay : Menu 2, in meters. "Y" coordinate of the transmitter dipole.

A-SP : Menu 2, in meters. Receiver dipole spacing.

Rx, Ry : Menu 4, in meters. Coordinates of the first receiver electrode.

Tx Curr : Menu 4, in amperes. Transmitter current.

N : Menus 3, 4. The product of a * N = potential electrode position with respect to Rx.

An example of a GRADIENT array setup:

The AB transmitter dipole is set up on grid line 52,000 N, a-spacing = 50 m, the north-south baseline runs through the middle of the array, so that A is at -350 meters (350 W) and B is at +350 meters (350 E). The first potential electrode as shown is located at -100 meters (100 W), and 100 meters north of the transmitter dipole, or 52100. The data would be input as follows:

MENU 1

Ax   = -350

Bx   = +350
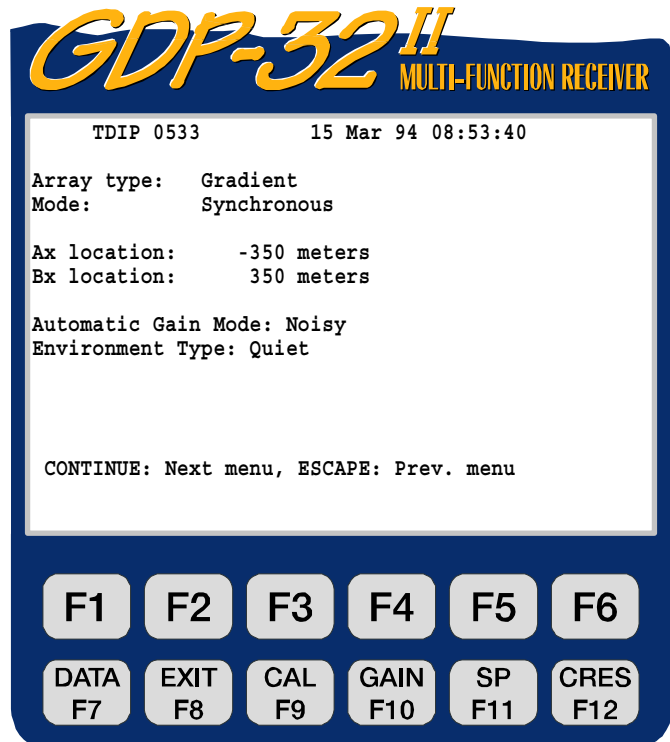
MENU 2

A-Sp = 50 m

Ay   = 52000

```
    TDIP 0533           15 Mar 94 08:53:45

OPERATOR    1  TX ID   1  A-SP      50
JOB 91001   Ay:   52000 N  SPREAD  1




CONTINUE: Next menu, ESCAPE: Prev. menu
```

F1 F2 F3 F4 F5 F6

| DATA F7 | EXIT F8 | CAL F9 | GAIN F10 | SP F11 | CRES F12 |

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```
0094 TDIP 0533    12.3 15 Mar 94 08:57:59
Survey Grd
Ry   52100 Rx    -100 N OUT
.125 Hz      /   32 Cycle  Tx Curr      1

CH       N
1 ON     1
2 ON     2
3 ON     3
4 OFF
5 OFF
6 OFF
7 OFF
8 OFF
CONT-Take data, ESC-Prev Menu
```

F1 F2 F3 F4 F5 F6

| DATA F7 | EXIT F8 | CAL F9 | GAIN F10 | SP F11 | CRES F12 |

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MENU 4

<—   We selected the Gradient array

Ry   = 52100

Rx   = -100

SCHLUMBERGER

```
                    ┌──────────────────┐
            ┌───────┤   Transmitter    ├───────┐
            │       └────────┬─────────┘       │
            │   ┌─────────┬──┴──┬─────────┐     │
            │   │  ┌──────┤ Rcvr├──────┐  │     │
            │   │  │   ┌──┴─────┴──┐   │  │     │
            │   │  │   │ ←─  a  ─→ │   │  │     │
            │   │  │   └───────────┘   │  │     │

    Ax        m₃  m₂  m₁         n₁ n₂ n₃        Bx
```
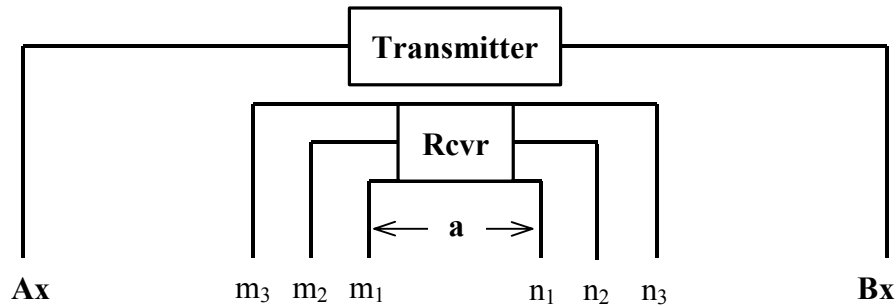
mA = ½ (AB- Na)

$$RHO = 2\pi * \frac{V}{I} * \cfrac{1}{\cfrac{1}{mA} - \cfrac{1}{mB} - \cfrac{1}{nA} + \cfrac{1}{nB}}$$

mB = ½ (AB+Na)

nA = ½ (AB+Na)

nB = ½ (AB- Na)

Where: a = [**A-SP**] = a-spacing (size of receiver dipole) in meters

V = Received voltage in volts [Measured].

I = 4/π * [**Tx Curr**] = 4/π * Transmitter current in amperes

N = multiplier for potential dipole length. Used as N-spacing in other programs.

m,n = potential electrode locations.

A,B = current electrode locations. Ax and Bx are input in menu 1, in meters, and can be coordinate locations.

AB = |Ax - Bx| in meters.

Assumption: Receiver dipoles are centered about the middle of the transmitter (AB) dipole. Note that the Schlumberger and Wenner arrays are special cases of the Gradient array.

Input Parameters:

Ax, Bx : Menu 1, in meters.

A-SP   : Menu 2, in meters. Shown as "**a**" in the equations.

N   : Menus 3 and 4. The product of **a * N** = the potential dipole length.

Tx Curr : Menu 4, in amperes

In the above diagram, if the potential dipoles are 10, 20 and 30 meters long, with a = 10 m, and the 10 m dipole is connected to channel 1, the 20 m dipole to channel 2 and the 30 m dipole connected to channel 3, the operator would enter N=1 for channel 1, N=2 for channel 2, and N=3 for channel 3.

LINE number in Menu 2 is used by data processing programs as a grid coordinate, in a direction perpendicular to the survey line.

The Tx and Rx fields are used as survey parameters by data processing programs. Whenever Menu 1 is accessed, the Ax and Bx values will automatically set Tx = AB/2 and Rx = Ax + AB/2 = receiver location.

The operator may change the AB/2 value in the Tx field while in Menu 4, rather than returning to Menu 1, in order to increase the survey speed. The Rx value will not be changed during a sounding, unless the operator returns to Menu 1, resulting in Tx and Rx being automatically updated.

The data acquisition program will always use the value of Tx = AB/2 for resistivity calculations.

An example of a SCHLUMBERGER setup:

Let the AB transmitter dipole be 1000 meters long, the starting potential dipole spacing = 10 m, and we want to measure 3 potential dipoles simultaneously with logarithmic spacing: 10 m, 30 m and 100 m.

The data would be input as follows:

Menu 1:		Menu 2:	Menu 4:	
Ax	= **1000**	A-Sp = **10 meters**	CH	N
Bx	= **0**		1 ON	**1**
			2 ON	**3**
			3 ON	**10**

An example of a WENNER setup using the Schlumberger program:

Let the AB transmitter dipole be 90 meters long, then the potential dipole spacing will be 30 m.

The data would be input as follows:

Menu 1:		Menu 2:	Menu 4:	
Ax	= **90**	A-Sp = **30 meters**	CH	N
Bx	= **0**		1 ON	**1**

15.5   USEFUL CSAMT EQUATIONS

Cagniard Resistivity, ρ_a:

$$\rho_a = \frac{1}{5f}\left|\frac{E}{H}\right|^2_{\perp} \qquad \text{(ohm-meters)}$$

Phase difference, Φ :

$$\phi = \phi E - \phi H \qquad \text{(milliradians)}$$

Skin depth δ is defined by:

$$\delta = \sqrt{\frac{2}{\mu\ \sigma\ \omega}} = 503\sqrt{\frac{\rho_a}{f}} \qquad \text{(meters)}$$

Equivalent depth of investigation, D:

$$D = 356\sqrt{\frac{\rho_a}{f}} \qquad \text{(D in meters)}$$

Wavelength, λ:

$$\lambda = 2\pi\delta$$

where:

E = electric field in mV/km

H = magnetic field in gamma (γ) or nanoteslas (nT)

Φ = phase in milliradians

μ = magnetic permeability of air

= 4 π x 10^{-7}

σ = conductivity in siemen/meter

= $1/\rho$

$\omega = 2\ \pi\ f$,

f=frequency in Hz

ρ = resistivity in ohm-meters

= $1/\sigma$

Far Field CSAMT Depth of Investigation
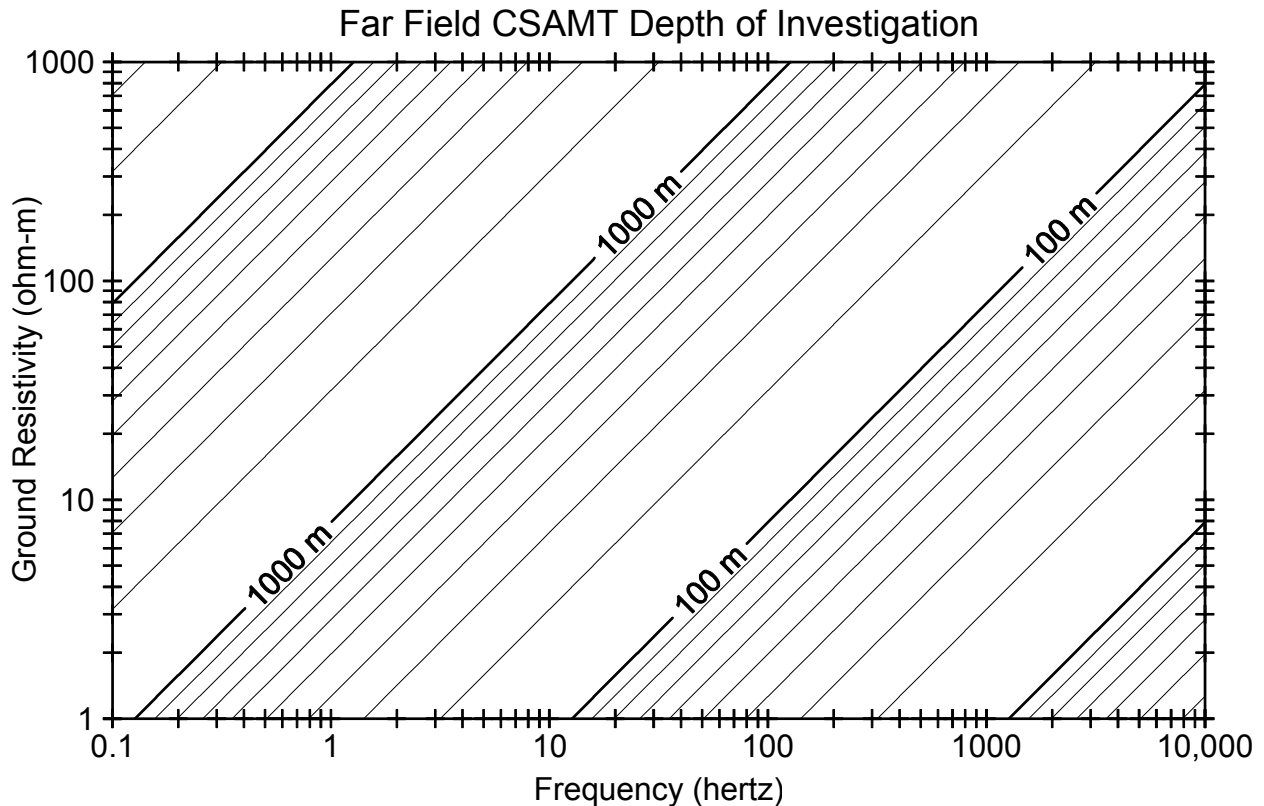
15.6   USEFUL TEM EQUATIONS

Apparent Resistivity, ρ_a, (late time)

$$\rho_a = \left(\frac{I \quad A_T \quad A_R}{V} \right)^{(2/3)} \left(\frac{1}{t} \right)^{\frac{5}{3}} \times 6.3219 E - 3 \qquad \text{(ohm-meters)}$$

Diffusion depth, d:

$$d = 40 \sqrt{\frac{t}{\sigma}} = 40 \sqrt{\rho_a t} \quad \text{(meters)}$$

Equivalent depth of investigation, D:

$$D = 28 \sqrt{\frac{t}{\sigma}} = 28 \sqrt{\rho_a t} \quad \text{(meters)}$$

where:

A_T = transmitter moment in square meters

A_R = Receiver moment in square meters

I   = Transmitter current in ampere   t   = Time in milliseconds

V   = Received voltage in microvolts σ = Conductivity in siemens/meter = $1/\rho$

Depth of investigation plot for TEM assuming a noise level of 1 nanovolt per square meter, showing the absolute limits for 100 by 100 and 500 by 500 m loops.